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Spectral analysis of malfunction mode in endmilling

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Abstract

For the investigation of the chatter modes, the power spectrum of the parametric time series model was adopted and analyzed at several mixed conditions of different revolution. This paper describes a methodology for an application of several time series such as *AR* (forward-backward, burg, least square, Yule Walker, geometric lattice, instrumental variable), *ARX* (least square, instrumental variable), *ARMAX*, *ARMA*, *Box Jenkins*, *Output Error*. To estimate the chatter mode using their spectral analysis their results were compared with one another. As a result, it was proven that several time series methods can be used for chatter mode estimation. Among them, the *ARX*, *ARMAX* and instrumental variable methods (*iv4*) are more desirable and reliable than the other algorithm for the exact calculation of the chatter mode in endmilling. Among three cutting forces, the z direction cutting force, *Fz*, has more powerful characteristics of chatter occurring than the cutting forces, *Fx* and *Fy*, in the sense that weak mode is calculated exactly and there is no shifted or pseudo mode in the estimated power spectra of endmilling forces.

Keywords: Chatter; Power spectrum; Time series modelling; Transfer function

1. Introduction

Increasing interest and its results on chatter mechanics have evolved over the past decades. Dilley et al. showed the effect of chisel edge on chatter frequency using *FFT* (Fast Fourier Transform) in drilling [1]. Insperger et al. described the analytical and experimental identification of the chatter frequencies in milling and the frequency diagram are attached to the stability charts [2]. Kim et al. also showed the chatter prediction method [3]. With the advancement of a chatter monitoring, the conventional *FFT* method was generally used. But it needs huge number of data for reliable monitoring. Also it is difficult to identify a system equation for later approach. To investigate the chatter dynamic mechanism between the endmill and workpiece, it is essential to identify the chatter dynamics at first for calculating chatter frequency and its damping ratio. Among several identification methods, the time series parametric modelling is an adequate method in the sense that it satisfies these needs. In this study, several algorithms are analyzed and discussed with one and another. Also they were extended to a spectral analysis comparing with *FFT*. Using these time series, a weak chatter frequency is detected easily in endmilling. It also shows precise chatter mode that was calculated directly from the part of auto regressive parameter. The chatter modes calculated are consistent well with each other, but some algorithms have some drawbacks. The chatter mode was estimated mathematically using several algorithms such as *AR*, *ARMA*, *ARX*, *ARMAX*, *BJ* and *OE*

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[4-14]. The advantages and limitations of each algorithm were compared and the behaviours of chatter modes are also discussed. The reliability of several algorithms using the cutting forces in three directions was discussed in this study. In the model considering the moving average input, a noise input was used for the modelling input.

2. Time series algorithms

2.1 ARMA and AR model

The general formulation of an *ARMA(n,m)* (*ARMA*: Auto Regressive Moving Average) model is considered for the endmilling and its process may be defined as *ARMA(n,m)*.

$$(1 + a_1 z^{-1} + a_2 z^{-2} + \quad \ldots \quad + a_n z^{-n}) y(t) = (1 + b_1 z^{-1} + b_2 z^{-2} + \quad \ldots \quad + b_m z^{-m}) e(t) \quad \text{or}$$

$$A(z^{-1}) y(t) = B(z^{-1}) e(t) \tag{1}$$

where $y(t)$ is defined as the output data, a_i is the autoregressive parameter, b_i is moving average parameter and the integers *n and m* are the order of the autoregressive and moving average parameters, respectively. Also z^{-1} is the backshift operator such as $z^{-1}y(t) = y(t-1)$ and $e(t)$ is assumed to be white noise. If the order m equals zero, the *ARMA(n,m)* model may be reduced to the *AR(n)* model and can be identified using only autoregressive parameters. And it can be written in the following:

$$(1 + a_1 z^{-1} + a_2 z^{-2} + \quad \ldots \quad + a_n z^{-n}) y(t) = e(t) \quad \text{or}$$

$$A(z^{-1}) y(t) = e(t) \tag{2}$$

At this level, we can estimate using several methods such as forward-backward, burg, least square, Yule Walker, geometric lattice and instrument variable(iv) may be used for modelling and analysis [4-9].

2.2 ARMAX and ARX model

The *ARMAX(n,m,l,nk)* model is defined as follows: $A(z^{-1})$ is called an autoregressive parameter and the $B(z^{-1})$ and $C(z^{-1})$ are moving average ones.

$$(1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}) y(t) = (b_1 + b_2 z^{-1} + b_3 z^{-2} + \ldots + b_m z^{-m}) u(t - nk)$$

$$\text{or} \quad A(z^{-1}) y(t) = B(z^{-1}) u(t) + C(z^{-1}) e(t) \tag{3}$$

$$+ (1 + c_1 z^{-1} + c_2 z^{-2} + \ldots + c_l z^{-l}) e(t)$$

If the moving average part $C(z^{-1})$ of *ARMAX* becomes a unity by assuming that the past noise has no correlation with current one, then the *ARMAX* *(n,m,l,nk)* model is reduced to a *ARX(n,m,nk)* model:

$$(1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}) y(t) = (b_1 + b_2 z^{-1} + b_3 z^{-2} + \ldots + b_m z^{-m}) u(t - nk) + e(t) \tag{4}$$

2.3 Box Jenkins (BJ) model

The endmilling process may be modelled by assuming that the output has a different correlation between $u(t)$ and $e(t)$. It can be defined as the *Box Jenkins*, *BJ (n,m,l,k,nk)*:

$$y(t) = \frac{(b_1 + b_2 z^{-1} + \quad \ldots \quad + b_n z^{-n})}{(1 + f_1 z^{-1} + \quad \ldots \quad + f_l z^{-l})} u(t - nk) + \frac{(1 + c_1 z^{-1} + \quad \ldots \quad + c_m z^{-m})}{(1 + d_1 z^{-1} + \quad \ldots \quad + d_k z^{-k})} e(t) \tag{5}$$

where $B(z^{-1})$, $C(z^{-1})$, $D(z^{-1})$ and $F(z^{-1})$ are polynomial functions.

2.4 Output Error (OE) model

In the *BJ* model, if the part of $C(z^{-1})/D(z^{-1})$ equals unity by assuming that there is no correlation between the past noise and current one, the output error model, *OE(n,l,nk)*, can be obtained as follows.

$$y(t) = \frac{(b_1 + b_2 z^{-1} + \quad \ldots \quad + b_n z^{-n})}{(1 + f_1 z^{-1} + \quad \ldots \quad + f_l z^{-l})} u(t - nk) + e(t) \tag{6}$$

2.5 Natural mode and spectrum estimation

Using time series models, the chatter frequency and its damping ratio can be calculated from the denominator part of the transfer function of the models. By breaking up the denominator into factors, the terms such as $(1-\lambda_i z^{-1})(1-\lambda_i^* z^{-1})$ may be obtained and the chatter mode can be calculated using the Eqs. (7) and (8). If the sampling period is T_s, the chatter frequency and its damping ratio can be summarized as follows [4].

$$\varpi_i = \frac{1}{T_s}\sqrt{\frac{\left[\ln(\lambda_i\cdot\lambda_i^{*})^2\right]}{4}+\left[\cos^{-1}\left(\frac{\lambda_i+\lambda_i^{*}}{2\cdot\sqrt{\lambda_i\cdot\lambda_i^{*}}}\right)\right]^2} \quad (7)$$

$$\xi_i = \sqrt{\frac{\left[\ln(\lambda_i\cdot\lambda_i^{*})\right]^2}{\left[\ln(\lambda_i\cdot\lambda_i^{*})\right]^2+4\left[\cos^{-1}\left(\frac{\lambda_i+\lambda_i^{*}}{2\cdot\sqrt{\lambda_i\cdot\lambda_i^{*}}}\right)\right]^2}} \quad (8)$$

The transfer function of a model can be summarized in direct form as follows.

$$H(z^{-1})=\frac{N(z^{-1})}{D(z^{-1})}=\frac{(1+b_1z^{-1}+b_2z^{-2}+b_3z^{-3}+\ \dots\ +b_mz^{-m})}{(1+a_1z^{-1}+a_2z^{-2}+\ \dots\ +a_nz^{-n})} \quad (9)$$

or

$$H(z^{-1})=\frac{N(z^{-1})}{D(z^{-1})}=\frac{(b_1+b_2z^{-1}+b_3z^{-2}+\ \dots\ +b_{m+1}z^{-m})}{(1+a_1z^{-1}+a_2z^{-2}+\ \dots\ +a_nz^{-n})} \quad (10)$$

And the power spectrum can be calculated from Eqs. (9) and (10) by substituting $z^{-1} = e^{-iwT}$. The transfer functions may have real and imaginary part and by calculating the root of square sum of real and imaginary part, the power spectrum of the model can be calculated.

3. Experiments and discussion

3.1. Experimental methods

The experiment for getting cutting forces in endmilling was performed in a vertical machining center (DOOMAC 40V). The forces, *Fx*, *Fy* and *Fz*, were acquired using tool dynamometer at the conditions of 500 rpm ~ 1300 rpm, axial depth of 4 mm and feedrate of 0.02 mm/rev. The endmill used has a shape of φ15 mm × 48.7 mm (diameter × overhang) with four flutes. The acquired cutting force was used for the spectrum analysis of time series model. The visual c++ was used for data arrangement of re-sampling. The type of tool dynamometer is the piezo type (kistler: 9272B). The experimental set up is shown in Fig. 1. The machining method of the experiment is shown in Fig. 2. The workpiece was machined by considering five steps as in Fig. 2 for causing chatter at the right fifth stage (⑤) of the workpiece. The rotational speed changes from 500 rpm to 1300 rpm.

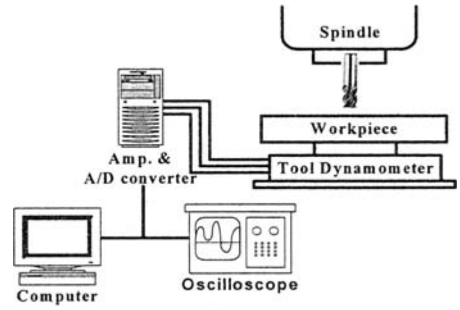

Fig. 1. Configuration of experimental set up.

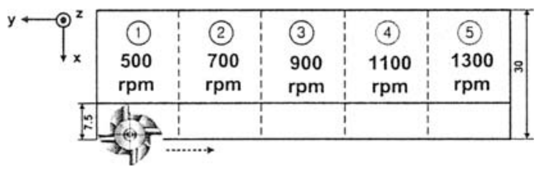

Fig. 2. Configuration of workpiece for experiments.

Table 1. Cutting condition for experiments.

Workpiece	SM45C	Radial depth	7.5 mm
Axial depth	4 mm	Cutting method	Up-milling
Endmill	Diameter = 15 mm, Helix angle = 30 °, four fluted HSS		
Rotational speed	stage ① : 500 rpm, stage ② : 700 rpm, stage ③ : 900 rpm		
	stage ④ : 1100 rpm, stage ⑤ : 1300 rpm		

The feedrate of 0.02 mm/rev was fixed at all stages from the first stage(①) to the fifth stage(⑤). In experiments, a chatter occurs at the fifth stage(⑤) around 20.5 sec. The other cutting conditions in this experiment are summarized in Table 1.

3.2 Mode estimation

Fig. 3 shows the cutting force in five stages (①-⑤) for cutting forces *Fx, Fy* and *Fz.* In these experiments the sampling frequency was selected as 5 kHz. The chatter frequency was detected using FFT. Also the power spectrum of times series model was acquired, at this time, the order of an autoregressive and moving average part was selected as an order of (10,5) (however n=5, m=3, l=10 and k=10 for *BJ* model). For a small order in moving average, it revealed some pseudo mode. So the order of a moving average parameter was selected as 5 for no mode shifting. The modes of a tooth passing frequency in endmilling (33Hz for 500 rpm, 47Hz for 700 rpm, 60 Hz for 900 rpm, 73 Hz for 1100 rpm and

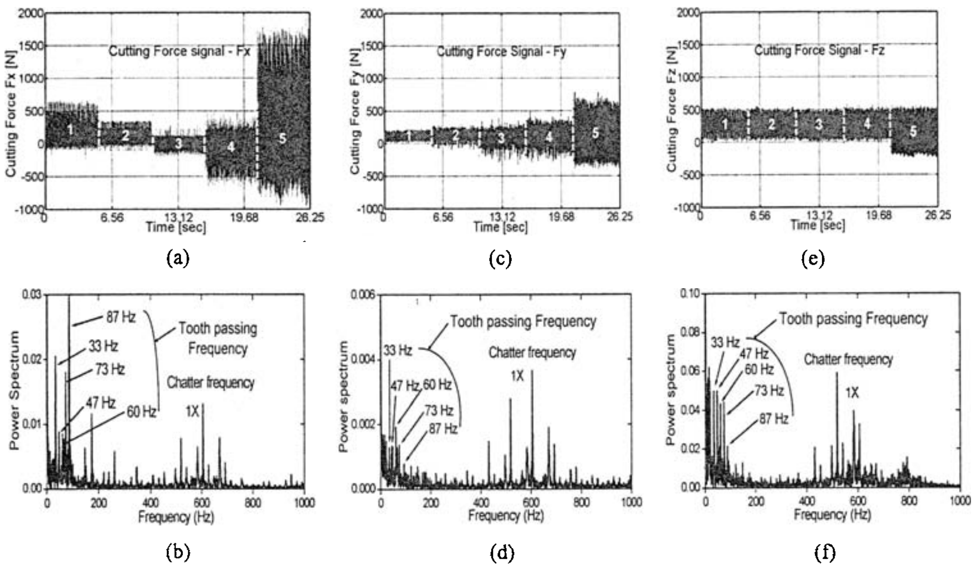

Fig. 3. Measured cutting force in three directions Fx(a), Fy(c) and Fz(e) and their power spectra (b),(d) and (f) using FFT.

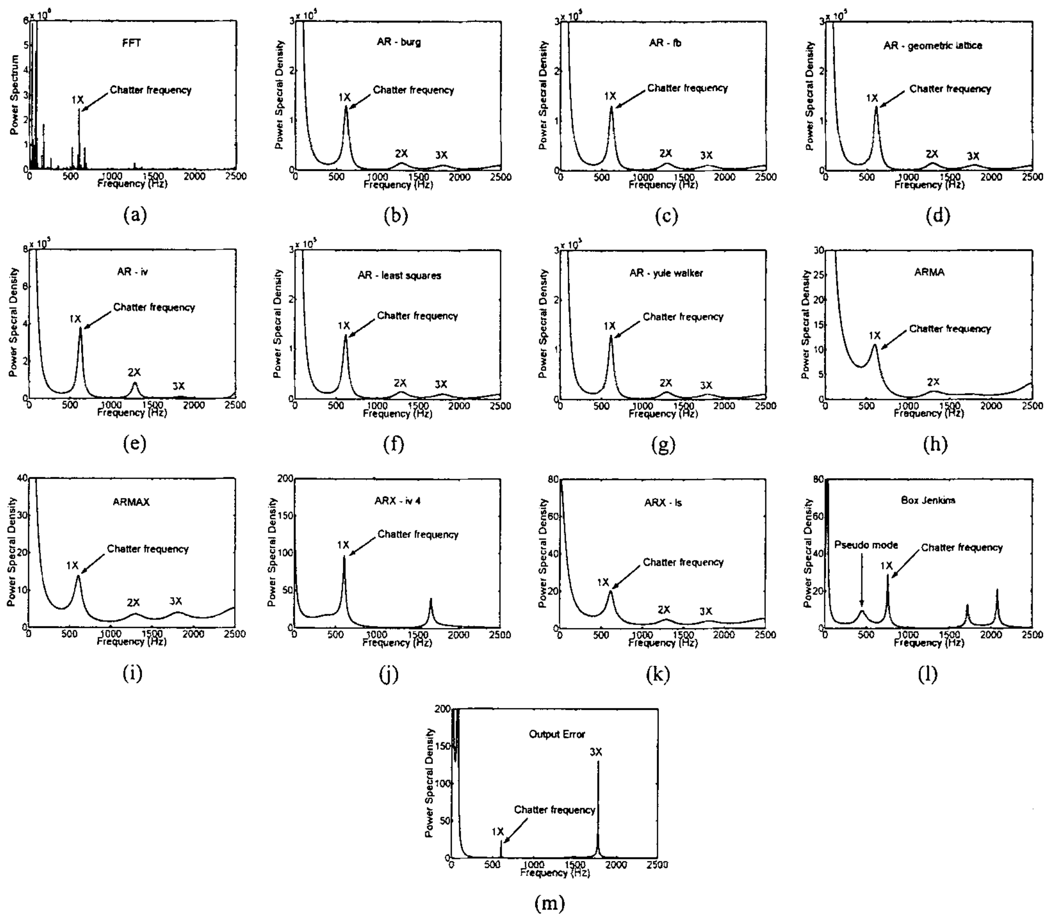

Fig. 4. Power spectra of cutting force Fx with the sampling frequency of 5 kHz.

87 Hz for 1300 rpm) were shown in the power spectrum of cutting forces, *Fx*, *Fy* and *Fz* (Fig. 3(b), (d) and (f)).

Fig. 4 shows the power spectrum of the cutting force *Fx* for Fig. 3(a). The harmonic tooth passing frequency was shown due to the teeth of an endmill in spectrum. The chatter was shown in the fifth stage of the cutting force and its chatter frequency appears around 1X (about 620~740 Hz) and its harmonic also appears at 2X (about 1293 Hz ~ 1800 Hz) in several algorithms respectively. To analyze vibrating property of the cutting force, the sampling frequency was set to 5 kHz. The amplitude of harmonic components in FFT decreases in the higher frequency range compared to that of the first dominant frequency, 1X. In the power spectrum of the time series (for *ARMAX*, *BJ*, *OE*, *Iv*4) the global trend is similar, but some weak mode is magnified and can be seen more clearly than *FFT*. Among the time series algorithms the *ARMAX* is most suitable to represent the exact chatter mode (i.e. 1X : ω_i = 611.13 Hz and ξ_i = 0.0557) in endmilling process. Furthermore, the chatter mode and its harmonic one (1X or 2X) may be obtained mathematically using the Eqs. (10) and (11). The other weak modes may also be calculated and shown in Table 2, which cannot be seen in spectrum of Fig. 4. The results are summarized in Table 2. Fig. 4 (a) shows the spectrum by *FFT* method. Fig. 4 (b)-(l) shows the spectra of each time series algorithms, i.e. *AR* ((b)-(g)), *ARX*(h), *ARMAX*(i), *BJ*(j), *OE*(k), *iv4*(l) respectively. The *AR*, *ARX*, *ARMAX* and *OE* model is reasonable method for chatter mode detection of 1X. But in *BJ* method, some pseudo and shifted modes appear at the spectrum in Fig. 4(l). Among *AR* models, *AR*(iv) intensifies the chatter mode, 1X, more than the harmonic mode, 2X. For the models (*ARX*, *ARMAX*, *BJ* and *OE* model) that need new input, the independent white noise was also used as an input *u(t)*. The mode shown in spectrum conforms well to the calculated modes and the result was summarized in Table 2.

Considering the power spectrum of the cutting force *Fy* and *Fz* for all stages ranging ① ~ ⑤ in Fig. 2(b), It also shows similar spectrum comparing with cutting force *Fx* that harmonics of the tooth passing components appears in the spectrum of *FFT* due to the teeth of tool rotation. The tooth passing frequency may be damped smoothly and around the natural mode of the endmill dynamics the chatter frequency appears clearly. So, the cutting force in *Fy* shows the

Table 2. Chatter frequency of endmilling dynamics using cutting force *Fx*.

Mode	1st	2nd	3rd	4th	5th
AR(10) (burg) (ω_i)	622.28	1293.7	1801.9		
(ζ_i)	0.0533	0.0848	0.0982		
AR(10) (fb)	622.32	1293.8	1801.9		
	0.0532	0.0847	0.0981		
AR(10) (gl)	622.28	1293.8	1801.9		
	0.0533	0.0848	0.0982		
AR(10) (iv)	1845.3	1289.9	630.41	93.470	
	0.0674	0.0379	0.0461	0.5121	
AR(10) (ls)	622.31	1293.8	1801.9		
	0.0532	0.0847	0.0982		
AR(10) (yw)	622.25	1293.8	1802.1		
	0.0534	0.0849	0.0983		
ARMA(10,3)	616.37	1297.4	1812.1	93.470	
	0.0714	0.0981	0.0996	0.5121	
ARMAX(10,5,3,0)	611.13	66.959	1297.2	1808.7	
	0.0557	0.2603	0.0985	0.1010	
ARX(10,3,0) (ls)	622.34	1293.8	1801.9	93.470	
	0.0533	0.0847	0.0982	0.5121	
ARX(10,3,0) (iv4)	1662.4	1966.8	608.00	386.50	
	0.0104	0.1754	0.0190	0.2420	
BJ(10,3,10,3,0) (c/d)	43.107	601.32	1300.8	1825.1	
	0.2492	0.0818	0.1209	0.1246	
BJ(10,3,10,3,0) (b/f)	2076.2	1717.5	760.43	453.14	30.319
	0.0062	0.0078	0.0089	0.0612	0.0047
OE (3,10,0)	1777.2	1466.8	605.39	72.783	30.319
	0.0009	0.0555	0.0005	0.0054	0.0047

same chatter in this signal but the second harmonic chatter frequency was magnified more than first one. The second harmonic frequency of cutting force *Fy* was magnified more than the cutting force *Fx*. The first chatter frequency was appeared around 1X (around 591.2 Hz for *ARMAX* ~ 594.8 Hz for *AR*). So the cutting force *Fy* also includes the chatter property and it can be used for the chatter detection. The second channel of tool dynamometer was used to acquire the cutting force *Fy*. In this case, the harmonic components 1X and 2X (about 1117 Hz for *ARMAX* ~ 1112 Hz for *AR*) are also generated. In a power spectrum of time series the amplitude of the following harmonic components decreases in higher frequency range comparing to that of the first synchronous frequency. But the global trend is similar and a weaker mode is magnified. This phenomenon can be seen clearly and the chatter mode 1X or its harmonics 2X in the spectrum may be calculated using the Eqs. (7) and (8). The calculated modes for *Fy* and *Fz* are summarized in Table 3. The estimated mode is very close to the real mode acquired by the *FFT*. The right side of Table 3 represents the natural

Table 3. Chatter frequency of endmilling dynamics using cutting force *Fy* and *Fz*.

mode	Fy				Fz			
	1st	2nd	3rd	4th	1st	2nd	3rd	4th
AR(10) (burg) (ω_i)	2239.4	1658.3	1112.3	594.86	2154.4	1577.5	558.48	846.72
(ζ_i)	0.1137	0.0739	0.0859	0.0513	0.1631	0.1228	0.0572	0.1750
AR(10) (fb)	2239.4	1658.3	1112.3	594.86	2154.4	1577.5	558.46	846.77
	0.1137	0.0739	0.0860	0.0513	0.1631	0.1228	0.0572	0.1750
AR(10) (gl)	2239.4	1658.3	1112.3	594.86	2154.4	1577.5	558.48	746.72
	0.1137	0.0739	0.0859	0.0513	0.1631	0.1228	0.0572	0.1750
AR(10) (iv)	594.18	1123.9	1628.0	594.86	2158.0	1603.4	527.53	792.58
	0.0864	0.1342	0.0521	0.0513	0.1646	0.0577	0.0548	0.0642
AR(10) (ls)	2239.4	1658.3	1112.3	594.86	2154.4	1577.5	558.45	846.78
	0.1137	0.0739	0.0859	0.0513	0.1631	0.1228	0.0571	0.1750
AR(10) (yw)	2239.3	1658.3	1112.3	594.87	2157.2	1578.5	558.66	847.38
	0.1137	0.0739	0.0860	0.0514	0.1638	0.1241	0.0580	0.1782
ARMA(10,3)	2248.5	1657.7	1117.0	597.67	2147.2	1574.2	561.90	842.71
	0.1212	0.0811	0.0856	0.0515	0.1554	0.1309	0.0674	0.1703
ARMAX(10,5,3,0)	591.20	1153.6	1635.9	2205.2	2278.0	1590.6	764.23	535.94
	0.0619	0.0887	0.0423	0.1884	0.1222	0.1669	0.1463	0.0471
ARX(10,3,0) (ls)	2239.5	1658.3	1112.3	574.89	2154.5	1577.6	558.40	846.72
	0.1139	0.0740	0.0860	0.0512	0.1634	0.1228	0.0572	0.0175
ARX(10,3,0) (iv4)	1411.8	2028.1	1372.0	606.35	2260.4	651.42	955.49	1875.7
	0.3570	0.0551	0.0448	0.0706	0.0383	0.0436	0.1374	0.3793
BJ(10,3,10,3,0) (c/d)	596.50	1144.3	1649.1	2217.1	2247.1	1588.2	773.76	540.65
	0.0685	0.0902	0.0556	0.1663	0.1211	0.1866	0.1280	0.0560
BJ(10,3,10,3,0) (b/f)	593.08	1727.0	876.30	1321.8	2082.5	1073.7	649.40	540.65
	0.0287	0.1116	0.0724	0.0152	0.0018	0.0034	0.0011	0.0560
OE (3,10,0)	1950.3	1291.7	605.45	1321.5	2031.2	1160.9	677.09	437.54
	0.0515	0.0003	0.0013	0.4627	0.0696	0.0210	0.0339	0.1294

mode of *Fz*. By decreasing the sampling frequency a lower frequency range can be analyzed exactly with a high resolution. In this paper, the chatter mode of each model was calculated with the order, i.e. autoregressive $n = 10$ and moving average $m = 5$ for *AR, ARX, ARMAX* model but $n = 5$, $m = 3$, $l = 10$ and $k = 10$ for *BJ* model or *OE* model. The *AR, ARX, ARMAX* and *OE* model was proven to be reasonable methods for chatter analysis. But in the *BJ* method, a pseudo mode (about 876 Hz) appears in the spectrum. So it has some drawback. A higher order is needed for the multi mode. So the order of the model of autoregressive part is selected as ten for the estimation of five modes even if some error may be caused. However the error is negligible in our detail investigation of the spectrum. The five natural modes can be obtained by equating the characteristic equation equal to zero. The *ARMAX* are most desirable models in general for less data and weak characteristics of the cutting force. In some modelling, the *instrumental variable* algorithm and *BJ* also give good results for the calculation of chatter mode. But the latter has a more possibility of pseudo modes. The chatter and other modes of frequency and damping ratio can be estimated similarly by using Eqs. (7) and (8) as in Table 3. If a lower order model is selected it is very difficult to discriminate the two close modes exactly. These characteristics result in a smoothing effect of the weak modes that are merged into stronger one. It is also very important to select the sampling frequency and the time series algorithms appropriately. By avoiding the aliasing effect the *ARMAX* is most desirable methods for the chatter mode analysis in an endmilling with no shifting of a real mode.

4. Conclusions

1. Several time series algorithms have been introduced and chatter mode and power spectrum estimation in an endmilling was accomplished. The advantages and drawbacks of time series modelling can be verified by analyzing the characteristics of the power spectrum. These results may be used for the estimation of chatter frequency that happens frequently in real endmilling.

2. The *ARMAX* model was proven to be most suitable for the estimation of malfunction and it can

be used for obtaining the chatter mode with less error. The first chatter mode (1X) located at around 670 Hz and damping factors were calculated with less bias. The time series is also better methods to obtain the chatter mode by modelling with a lower sampling rate and it can discriminate the real close chatter modes by using Eqs. (7) and (8).

3. Using the cutting force *Fy* and *Fz*, powerful characteristics of a chatter mode appear. They do not show shifting or pseudo mode in the power spectrum. The cutting force in z-direction reveals chatter mode most strongly among three cutting forces.

Nomenclature

a_i	: autoregressive parameter of *ARMA* model
$y(t)$	: observation at time t
b_i, c_i	: moving average parameter of *ARMA* model
A_o	: residual sum of the higher order model
d_i, f_i	: autoregressive parameter of BJ or *OE* model
A_1	: residual sum of the lower order model
$e(t)$	: white noise
AIC	: Akaike information criterion
fb	: forward - backward
AR	: autoregressive model
gl	: geometric lattice
ARMA	: autoregressive moving average model
ls	: least square
ARX	: autoregressive model with an input of new moving average part
t	: time variable
ARMAX	: autoregressive moving average model with an input of new moving average part
yw	: Yule walker

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